

ON ϕ -SYMMETRIC LP-SASAKIAN MANIFOLDS ADMITTING SEMI-SYMMETRIC METRIC CONNECTION

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ABSTRACT. The object of the present paper is to study locally ϕ -symmetric LP-Sasakian manifolds admitting semi-symmetric metric connection and obtain a necessary and sufficient condition for a locally ϕ -symmetric LP-Sasakian manifold with respect to semi-symmetric metric connection to be locally ϕ -symmetric LP-Sasakian manifold with respect to Levi-Civita connection.

1. INTRODUCTION

On the analogy of Sasakian manifolds, in 1989 Matsumoto [12] introduced the notion of LP-Sasakian manifolds. Again the same notion was studied by Mihai and Rosca [13] and they obtained many results. LP-Sasakian manifolds are also studied by De et. al. [8], Shaikh et. al. ([16], [17], [18], [20]), Taleshian and Asghari [28], Venkatesha and Bagewadi [29] and many others. The notion of locally ϕ -symmetry on a 3-dimensional LP-Sasakian manifold was studied by Shaikh and De [21].

In 1924 Friedmann and Schouten [10] introduced the notion of semi-symmetric linear connection on a differentiable manifold. Then in 1932 Hayden [11] introduced the idea of metric connection with torsion on a Riemannian manifold. A systematic study of the semi-symmetric metric connection on a Riemannian manifold has been given by Yano [30] in 1970. Also semi-symmetric metric connection on a Riemannian manifold has been studied by Barua and Mukhopadhyay [1], Binh [3], Chaki and Chaki [5], Chaturvedi and Pandey [6], Shaikh and Hui [23], Sharfuddin and Hussain [25] and many others. Recently Shaikh and Jana studied the quarter-symmetric metric connection on a (k, μ) -contact metric manifold [24].

The study of Riemann symmetric manifolds began with the work of Cartan [4]. A Riemannian manifold (M^n, g) is said to be locally symmetric due to Cartan [4] if its curvature tensor R satisfies the relation $\nabla R = 0$, where ∇ denotes the operator of covariant differentiation with respect to the metric tensor g . As a weaker version of local symmetry, the notion of locally ϕ -symmetric Sasakian manifold was introduced by Takahashi [27]. In the sense of Takahashi, Shaikh and Baishya [16] studied locally ϕ -symmetric LP-Sasakian manifolds. The notion of locally ϕ -symmetric manifolds in

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different structures has been studied by several authors (see, [7], [16], [19], [22], [27]). An LP-Sasakian manifold is said to be ϕ -symmetric [7] if it satisfies the condition

$$(1.1) \quad \phi^2((\nabla_W R)(X, Y)Z) = 0$$

for arbitrary vector fields X, Y, Z and W on M .

In particular, if X, Y, Z, W are horizontal vector fields, i.e., orthogonal to ξ , then it is called locally ϕ -symmetric LP-Sasakian manifold [27].

It is easy to check that an LP-Sasakian manifold is ϕ -symmetric if and only if it is locally symmetric or locally ϕ -symmetric.

Recently De and Sarkar [9] studied ϕ -Ricci symmetric Sasakian manifolds. In this connection Shukla and Shukla [26] studied ϕ -Ricci symmetric Kenmotsu manifolds. An LP-Sasakian manifold is said to be ϕ -Ricci symmetric [9] if it satisfies

$$(1.2) \quad \phi^2((\nabla_X Q)(Y)) = 0,$$

where Q is the Ricci operator, i.e., $g(QX, Y) = S(X, Y)$ for all vector fields X, Y .

If X, Y are horizontal vector fields then the manifold is said to be locally ϕ -Ricci symmetric.

It is easy to check that an LP-Sasakian manifold is ϕ -Ricci symmetric if and only if it is Ricci symmetric or locally ϕ -Ricci symmetric.

The object of the present paper is to study the locally ϕ -symmetric and locally ϕ -Ricci symmetric LP-Sasakian manifolds admitting semi-symmetric metric connection. The paper is organized as follows. Section 2 is concerned with some preliminaries about LP-Sasakian manifolds and semi-symmetric metric connection. Section 3 is devoted to the study of locally ϕ -symmetric LP-Sasakian manifolds admitting semi-symmetric metric connection and obtained a necessary and sufficient condition for a locally ϕ -symmetric LP-Sasakian manifold with respect to semi-symmetric metric connection to be locally ϕ -symmetric LP-Sasakian manifold with respect to Levi-Civita connection. Section 4 deals with the study of locally ϕ -Ricci symmetric LP-Sasakian manifolds admitting semi-symmetric metric connection.

2. PRELIMINARIES

An n -dimensional smooth manifold M is said to be an LP-Sasakian manifold ([13], [17]) if it admits a $(1, 1)$ tensor field ϕ , a unit timelike vector field ξ , an 1-form η and a Lorentzian metric g , which satisfy

$$(2.1) \quad \eta(\xi) = -1, \quad g(X, \xi) = \eta(X), \quad \phi^2 X = X + \eta(X)\xi,$$

$$(2.2) \quad g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \quad \nabla_X \xi = \phi X,$$

$$(2.3) \quad (\nabla_X \phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi,$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g . It can be easily seen that in an LP-Sasakian manifold, the following relations hold:

$$(2.4) \quad \phi\xi = 0, \quad \eta \circ \phi = 0, \quad \text{rank } \phi = n - 1.$$

Again, if we take

$$\Omega(X, Y) = g(X, \phi Y)$$

for any vector fields X, Y , then the tensor field $\Omega(X, Y)$ is a symmetric (0,2) tensor field [12]. Also, since the vector field η is closed in an LP-Sasakian manifold, we have ([8], [12])

$$(2.5) \quad (\nabla_X \eta)(Y) = \Omega(X, Y), \quad \Omega(X, \xi) = 0$$

for any vector fields X and Y .

Let M be an n -dimensional LP-Sasakian manifold with structure (ϕ, ξ, η, g) . Then the following relations hold ([16], [17]):

$$(2.6) \quad R(X, Y)\xi = \eta(Y)X - \eta(X)Y,$$

$$(2.7) \quad \eta(R(X, Y)Z) = \eta(X)g(Y, Z) - \eta(Y)g(X, Z),$$

$$(2.8) \quad S(X, \xi) = (n - 1)\eta(X),$$

$$(2.9) \quad S(\phi X, \phi Y) = S(X, Y) + (n - 1)\eta(X)\eta(Y),$$

$$(2.10) \quad (\nabla_W R)(X, Y)\xi = \Omega(Y, W)X - \Omega(X, W)Y - R(X, Y)\phi W,$$

$$(2.11) \quad (\nabla_W R)(X, \xi)Y = \Omega(W, Z)X - g(X, Z)\phi W - R(X, \phi W)Z$$

for any vector fields X, Y, Z , where R is the curvature tensor of g .

Let M be an n -dimensional LP-Sasakian manifold and ∇ be the Levi-Civita connection on M . A linear connection $\tilde{\nabla}$ on M is said to be semi-symmetric if the torsion tensor τ of the connection $\tilde{\nabla}$

$$\tau(X, Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X, Y]$$

satisfies

$$(2.12) \quad \tau(X, Y) = \eta(Y)X - \eta(X)Y$$

for all $X, Y \in \chi(M)$; $\chi(M)$ being the Lie algebra of all smooth vector fields on M . A semi-symmetric connection $\tilde{\nabla}$ is called semi-symmetric metric connection if it further satisfies

$$(2.13) \quad \tilde{\nabla}g = 0.$$

A semi-symmetric metric connection $\tilde{\nabla}$ in an LP-Sasakian manifold is defined by ([25],[30]):

$$(2.14) \quad \tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X, Y)\xi.$$

If R and \tilde{R} are respectively the curvature tensor of the Levi-Civita connection ∇ and the semi-symmetric metric connection $\tilde{\nabla}$ in an LP-Sasakian manifold, then we have [15]

$$(2.15) \quad \begin{aligned} \tilde{R}(X, Y)Z &= R(X, Y)Z - \alpha(Y, Z)X + \alpha(X, Z)Y \\ &\quad - g(Y, Z)LX + g(X, Z)LY, \end{aligned}$$

where α is a symmetric (0,2) tensor field given by

$$(2.16) \quad \alpha(X, Y) = (\tilde{\nabla}_X \eta)(Y) + \frac{1}{2}g(X, Y),$$

$$(2.17) \quad LX = \tilde{\nabla}_X \xi + \frac{1}{2}X = \phi X - \frac{1}{2}X - \eta(X)\xi$$

and

$$(2.18) \quad g(LX, Y) = \alpha(X, Y).$$

Lemma 2.1. [15] *In an LP-Sasakian manifold with semi-symmetric metric connection $\tilde{\nabla}$, we have*

$$(2.19) \quad \tilde{R}(X, Y)Z + \tilde{R}(Y, Z)X + \tilde{R}(Z, X)Y = 0,$$

$$(2.20) \quad g(\tilde{R}(X, Y)Z, U) = -g(\tilde{R}(Y, X)Z, U),$$

$$(2.21) \quad g(\tilde{R}(X, Y)Z, U) = -g(\tilde{R}(X, Y)U, Z),$$

$$(2.22) \quad g(\tilde{R}(X, Y)Z, U) = g(\tilde{R}(Z, U)X, Y).$$

Lemma 2.2. [15] *In an n -dimensional LP-Sasakian manifold the Ricci tensor \tilde{S} and scalar curvature \tilde{r} with respect to semi-symmetric metric connection $\tilde{\nabla}$ are given by*

$$(2.23) \quad \tilde{S}(X, Y) = S(X, Y) - (n-2)\alpha(X, Y) - ag(X, Y)$$

and

$$(2.24) \quad \tilde{r} = r - 2(n-1)a,$$

where $a = \text{tr. } \alpha$, S and r denote the Ricci tensor and scalar curvature of Levi-Civita connection ∇ respectively.

Lemma 2.3. [15] *Let M be an n -dimensional LP-Sasakian manifold with semi-symmetric metric connection $\tilde{\nabla}$. Then we have*

$$(2.25) \quad g(\tilde{R}(X, Y)Z, \xi) = \eta(\tilde{R}(X, Y)Z) = (\tilde{\nabla}_X \eta)(Z)\eta(Y) - (\tilde{\nabla}_Y \eta)(Z)\eta(X),$$

$$(2.26) \quad \tilde{R}(\xi, X)\xi = -\tilde{\nabla}_X \xi = X + \eta(X)\xi - \phi X,$$

$$(2.27) \quad \tilde{R}(X, Y)\xi = \eta(X)\tilde{\nabla}_Y \xi - \eta(Y)\tilde{\nabla}_X \xi,$$

$$(2.28) \quad \tilde{R}(\xi, X)Y = \eta(Y)\tilde{\nabla}_X \xi - g(Y, \tilde{\nabla}_X \xi)\xi,$$

$$(2.29) \quad \tilde{S}(X, \xi) = \left(\frac{n}{2} - a\right)\eta(X),$$

$$(2.30) \quad \begin{aligned} \tilde{S}(\phi X, \phi Y) &= S(X, Y) + \left(\frac{n}{2} - a\right)\eta(X)\eta(Y) \\ &\quad - (n - 2)\alpha(X, Y) - ag(X, Y) \end{aligned}$$

for arbitrary vector fields X, Y and Z .

From (2.2), (2.3), (2.5), (2.14) and (2.17), we get

$$(2.31) \quad \begin{aligned} (\tilde{\nabla}_W \tilde{R})(X, Y)\xi &= R(X, Y)W - R(X, Y)\phi W + \alpha(X, W)Y \\ &\quad - \alpha(Y, W)X + g(X, W)LY - g(Y, W)LX \\ &\quad + \alpha(Y, \phi W)X - \alpha(X, \phi W)Y + \Omega(Y, W)LX \\ &\quad - \Omega(X, W)LY + g(X, W)Y - g(Y, W)X \\ &\quad + g(Y, W)\phi X - g(X, W)\phi Y + \Omega(Y, W)X \\ &\quad - \Omega(X, W)Y + \Omega(X, W)\phi Y - \Omega(Y, W)\phi X \\ &\quad + \eta(X)[g(Y, W) - \Omega(Y, W)]\xi \\ &\quad - \eta(Y)[g(X, W) - \Omega(X, W)]\xi \end{aligned}$$

for arbitrary vector fields X, Y and W . Also from (2.14), (2.15) and (2.21), we have

$$(2.32) \quad g((\tilde{\nabla}_W \tilde{R})(X, Y)Z, U) = -g((\tilde{\nabla}_W \tilde{R})(X, Y)U, Z).$$

From (2.17) we have

$$(2.33) \quad \alpha(X, \xi) = \frac{1}{2}\eta(X),$$

$$(2.34) \quad (\nabla_W \alpha)(X, \xi) = \frac{1}{2}\Omega(W, X) - \alpha(X, \phi W),$$

$$(2.35) \quad \begin{aligned} (\nabla_W L)(X) &= [g(W, X) - \Omega(W, X)]\xi \\ &\quad + \eta(X)[W - \phi W] + 2\eta(X)\eta(W)\xi. \end{aligned}$$

Again by virtue of (2.33) - (2.35) we have from (2.14) and (2.15) that

$$\begin{aligned}
(2.36) \quad & (\tilde{\nabla}_W \tilde{R})(X, Y)Z \\
&= (\nabla_W R)(X, Y)Z - g(R(X, Y)Z, W)\xi + [g(W, Y) - \Omega(W, Y)]\eta(Z)X \\
&+ [g(W, Z) - \Omega(W, Z)]\eta(Y)X + 2\eta(Z)\eta(W)[\eta(Y)X - \eta(X)Y] \\
&+ \alpha(Y, Z)[g(X, W)\xi - \eta(X)W] + [\Omega(W, X) - g(W, X)]\eta(Z)Y \\
&+ [\Omega(W, Z) - g(W, Z)]\eta(X)Y + \alpha(X, Z)[\eta(Y)W - g(Y, W)\xi] \\
&- g(Y, Z)[\{g(W, X) - \Omega(W, X) - \alpha(X, W)\}\xi + \eta(X)\{\frac{1}{2}W - \phi W + 2\eta(W)\xi\}] \\
&+ g(X, Z)[\{g(W, Y) - \Omega(W, Y) - \alpha(Y, W)\}\xi + \eta(Y)\{\frac{1}{2}W - \phi W + 2\eta(W)\xi\}].
\end{aligned}$$

By virtue of (2.33) and (2.35) it follows from (2.14) that

$$\begin{aligned}
(2.37) \quad & (\tilde{\nabla}_X \tilde{S})(Y, Z) = (\nabla_X S)(Y, Z) - [S(X, Y) + \alpha(X, Y)]\eta(Z) \\
&+ [\frac{3}{2}g(X, Z) + (n-2)\Omega(X, Z)]\eta(Y) \\
&- (n-2)[g(X, Y) - \Omega(X, Y)]\eta(Z) - da(X)g(Y, Z).
\end{aligned}$$

Also from (2.8) we have

$$(2.38) \quad (\nabla_X S)(Y, \xi) = (n-1)\Omega(X, Y) - S(Y, \phi X).$$

3. LOCALLY ϕ -SYMMETRIC LP-SASAKIAN MANIFOLDS ADMITTING SEMI-SYMMETRIC METRIC CONNECTION

Definition 3.1. An LP-Sasakian manifold M is said to be locally ϕ -symmetric with respect to semi-symmetric metric connection if its curvature tensor \tilde{R} satisfies the condition

$$(3.1) \quad \phi^2((\tilde{\nabla}_W \tilde{R})(X, Y)Z) = 0$$

for all horizontal vector fields X, Y, Z and W .

We now consider a locally ϕ -symmetric LP-Sasakian manifold with respect to semi-symmetric metric connection. Then by virtue of (2.1) it follows from (3.1) that

$$(3.2) \quad (\tilde{\nabla}_W \tilde{R})(X, Y)Z + \eta((\tilde{\nabla}_W \tilde{R})(X, Y)Z)\xi = 0.$$

Using (2.32) in (3.2), we get

$$(3.3) \quad (\tilde{\nabla}_W \tilde{R})(X, Y)Z = g((\tilde{\nabla}_W \tilde{R})(X, Y)\xi, Z)\xi.$$

In view of (2.31) it follows from (3.3) that

$$\begin{aligned}
 (\tilde{\nabla}_W \tilde{R})(X, Y)Z &= \left[g(R(X, Y)W, Z) - g(R(X, Y)\phi W, Z) + \alpha(X, W)g(Y, Z) \right. \\
 (3.4) \quad &- \alpha(Y, W)g(X, Z) + g(X, W)\alpha(Y, Z) - g(Y, W)\alpha(X, Z) \\
 &+ \alpha(Y, \phi W)g(X, Z) - \alpha(X, \phi W)g(Y, Z) + \Omega(Y, W)\alpha(X, Z) \\
 &- \Omega(X, W)\alpha(Y, Z) + g(X, W)g(Y, Z) - g(Y, W)g(X, Z) \\
 &+ g(Y, W)\Omega(X, Z) - g(X, W)\Omega(Y, Z) + \Omega(Y, W)g(X, Z) \\
 &\left. - \Omega(X, W)g(Y, Z) + \Omega(X, W)\Omega(Y, Z) - \Omega(Y, W)\Omega(X, Z) \right] \xi
 \end{aligned}$$

for all horizontal vector fields X, Y, Z and W . Next, let us assume that in an LP-Sasakian manifold, the relation (3.4) holds for all horizontal vector fields X, Y, Z and W . Then it follows from (2.36) that (3.4) holds and consequently the manifold is locally ϕ -symmetric with respect to semi-symmetric metric connection. This leads to the following:

Theorem 3.1. *An LP-Sasakian manifold is locally ϕ -symmetric with respect to semi-symmetric metric connection if and only if the relation (3.4) holds for all horizontal vector fields X, Y, Z and W .*

In view of (2.32), it follows from (3.2) that

$$(3.5) \quad (\tilde{\nabla}_W \tilde{R})(X, Y)\xi = 0.$$

From (2.31) and (3.5) it follows that

$$\begin{aligned}
 (3.6) \quad &R(X, Y)W - R(X, Y)\phi W \\
 &= g(Y, W)X - g(X, W)Y + g(X, W)\phi Y - g(Y, W)\phi X \\
 &+ \Omega(X, W)Y - \Omega(Y, W)X + \Omega(Y, W)\phi X - \Omega(X, W)\phi Y \\
 &+ \alpha(Y, W)X - \alpha(X, W)Y + g(Y, W)LX - g(X, W)LY \\
 &+ \alpha(X, \phi W)Y - \alpha(Y, \phi W)X + \Omega(X, W)LY - \Omega(Y, W)LX
 \end{aligned}$$

for horizontal vector fields X, Y and W . Contracting (3.6), we get

$$\begin{aligned}
 (3.7) \quad S(Y, W) - S(Y, \phi W) &= (n - 1 + a - \psi)[g(Y, W) - \Omega(Y, W)] \\
 &+ (n - 2)[\alpha(Y, W) - \alpha(Y, \phi W)],
 \end{aligned}$$

where $\psi = \text{tr. } \Omega$ and $a = \text{tr. } \alpha$. Hence we can state the following:

Theorem 3.2. *In a locally ϕ -symmetric LP-Sasakian manifold with semi-symmetric metric connection the curvature tensor and the Ricci tensor are respectively given by (3.6) and (3.7).*

We now consider a locally ϕ -symmetric LP-Sasakian manifold with Levi-Civita connection. Then in [16], Shaikh and Baishya proved that

Theorem 3.3. *An LP-Sasakian manifold (M^n, g) is locally ϕ -symmetric with respect to Levi-Civita connection if and only if the following relation*

$$\begin{aligned}
 (3.8) \quad & (\nabla_W R)(X, Y)Z \\
 &= [2\{\Omega(Y, W)g(X, Z) - \Omega(X, W)g(Y, Z)\} + \Omega(Y, Z)g(X, W) - \Omega(X, Z)g(Y, W) \\
 &+ 2\{\Omega(Y, Z)\eta(X)\eta(W) - \Omega(X, Z)\eta(Y)\eta(W)\} - g(\phi R(X, Y)W, Z)]\xi \\
 &+ \eta(X)[\Omega(W, Z)Y - g(Y, Z)\phi W - R(Y, \phi W)Z] - \eta(Y)[\Omega(W, Z)X - g(X, Z)\phi W \\
 &- R(X, \phi W)Z] - \eta(Z)[2\{\Omega(Y, W)X - \Omega(X, W)Y\} - \phi R(X, Y)W - g(Y, W)\phi X \\
 &+ g(X, W)\phi Y] + 2\{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z)\eta(W).
 \end{aligned}$$

holds for arbitrary vector fields $X, Y, Z, W \in \chi(M)$.

Now we take a locally ϕ -symmetric LP-Sasakian manifold with respect to semi-symmetric metric connection. Then the relation (3.4) holds for any horizontal vector fields X, Y, Z, W .

Let X, Y, Z, W be arbitrary vector fields of $\chi(M)$. We now compute $(\tilde{\nabla}_{\phi^2 W} \tilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z$ in two different ways. Firstly by virtue of (2.1) it follows from (3.4) that

$$\begin{aligned}
 (3.9) \quad & (\tilde{\nabla}_{\phi^2 W} \tilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z \\
 &= [g(R(\phi^2 X, \phi^2 Y)\phi^2 W, \phi^2 Z) - g(R(\phi^2 X, \phi^2 Y)\phi^3 W, \phi^2 Z) \\
 &+ \alpha(\phi^2 X, \phi^2 W)\{g(Y, Z) + \eta(Y)\eta(Z)\} - \alpha(\phi^2 Y, \phi^2 W)\{g(X, Z) + \eta(X)\eta(Z)\} \\
 &+ \alpha(\phi^2 Y, \phi^2 Z)\{g(X, W) + \eta(X)\eta(W)\} - \alpha(\phi^2 X, \phi^2 Z)\{g(Y, W) + \eta(Y)\eta(W)\} \\
 &+ \alpha(\phi^2 Y, \phi^3 W)\{g(X, Z) + \eta(X)\eta(Z)\} - \alpha(\phi^2 X, \phi^3 W)\{g(Y, Z) + \eta(Y)\eta(Z)\} \\
 &+ \Omega(Y, W)\alpha(\phi^2 X, \phi^2 Z) - \Omega(X, W)\alpha(\phi^2 Y, \phi^2 Z) + \{g(X, W) + \eta(X)\eta(W)\}\{g(Y, Z) \\
 &+ \eta(Y)\eta(Z)\} - \{g(Y, W) + \eta(Y)\eta(W)\}\{g(X, Z) + \eta(X)\eta(Z)\} + \{g(Y, W) \\
 &+ \eta(Y)\eta(W)\}\Omega(X, Z) - \{g(X, W) + \eta(X)\eta(W)\}\Omega(Y, Z) \\
 &+ \{g(X, Z) + \eta(X)\eta(Z)\}\Omega(Y, W) - \{g(Y, Z) + \eta(Y)\eta(Z)\}\Omega(X, W) \\
 &+ \Omega(X, W)\Omega(Y, Z) - \Omega(Y, W)\Omega(X, Z)]\xi.
 \end{aligned}$$

From (2.4) we have

$$(3.10) \quad g(\phi^2 X, \xi) = g(\phi^2 Y, \xi) = g(\phi^2 Z, \xi) = 0$$

and hence $\phi^2 X, \phi^2 Y, \phi^2 Z$ are horizontal vector fields of $\chi(M)$. Then by virtue of (2.1) it follows that

$$\begin{aligned}
 (3.11) \quad & R(\phi^2 X, \phi^2 Y)\phi^2 W = R(X, Y)W + \{\eta(Y)X - \eta(X)Y\}\eta(W) \\
 &+ \{g(Y, W)\eta(X) - g(X, W)\eta(Y)\}\xi,
 \end{aligned}$$

$$(3.12) \quad R(\phi^2 X, \phi^2 Y)\phi^3 W = R(X, Y)\phi W + \{\Omega(Y, W)\eta(X) - \Omega(X, W)\eta(Y)\}\xi,$$

$$(3.13) \quad \alpha(\phi^2 X, \phi^2 W) = \alpha(X, W) + \frac{1}{2}\eta(X)\eta(W).$$

In view of (3.11) - (3.13), (3.9) yields

$$(3.14) \quad \begin{aligned} & (\tilde{\nabla}_{\phi^2 W} \tilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z \\ &= [g(R(X, Y)W, Z) - g(R(X, Y)\phi W, Z) + \alpha(X, W)\{g(Y, Z) + \eta(Y)\eta(Z)\} \\ &- \alpha(Y, W)\{g(X, Z) + \eta(X)\eta(Z)\} + \frac{1}{2}\{\eta(X)g(Y, Z) - \eta(Y)g(X, Z)\}\eta(W) \\ &+ \alpha(Y, Z)g(X, W) - \alpha(X, Z)g(Y, W) + \frac{1}{2}\{\eta(Y)g(X, W) - \eta(X)g(Y, W)\}\eta(Z) \\ &+ \{\eta(X)\alpha(Y, Z) - \eta(Y)\alpha(X, Z)\}\eta(W) + \alpha(Y, \phi W)g(X, Z) - \alpha(X, \phi W)g(Y, Z) \\ &+ \{\eta(X)\alpha(Y, \phi W) - \eta(Y)\alpha(X, \phi W)\}\eta(Z) + \Omega(Y, W)\alpha(X, Z) - \Omega(X, W)\alpha(Y, Z) \\ &+ \frac{1}{2}\{\eta(X)\Omega(Y, W) - \eta(Y)\Omega(X, W)\}\eta(Z) + g(X, W)g(Y, Z) - g(Y, W)g(X, Z) \\ &+ g(Y, W)\Omega(X, Z) - g(X, W)\Omega(Y, Z) + \{\eta(Y)\Omega(X, Z) - \eta(X)\Omega(Y, Z)\}\eta(W) \\ &+ \Omega(Y, W)g(X, Z) - \Omega(X, W)g(Y, Z) + \{\eta(X)\Omega(Y, W) - \eta(Y)\Omega(X, W)\}\eta(Z) \\ &+ \Omega(X, W)\Omega(Y, Z) - \Omega(Y, W)\Omega(X, Z)]\xi. \end{aligned}$$

By virtue of (2.1) we have

$$(3.15) \quad \begin{aligned} (\tilde{\nabla}_{\phi^2 W} \tilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z &= (\tilde{\nabla}_W \tilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z \\ &+ \eta(W)(\tilde{\nabla}_\xi \tilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z. \end{aligned}$$

Now for any horizontal vector fields X, Y and Z we have from (3.4) that

$$(3.16) \quad (\tilde{\nabla}_\xi \tilde{R})(X, Y)Z = 0,$$

which implies that

$$(3.17) \quad (\tilde{\nabla}_\xi \tilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z = 0.$$

Using (3.17) in (3.15) we obtain

$$(3.18) \quad (\tilde{\nabla}_{\phi^2 W} \tilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z = (\tilde{\nabla}_W \tilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z.$$

In view of (2.1), we have

$$(3.19) \quad \begin{aligned} & (\tilde{\nabla}_W \tilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z \\ &= (\tilde{\nabla}_W \tilde{R})(X, Y)Z + \eta(Z)(\tilde{\nabla}_W \tilde{R})(X, Y)\xi \\ &+ \eta(Y)(\tilde{\nabla}_W \tilde{R})(X, \xi)Z + \eta(Y)\eta(Z)(\tilde{\nabla}_W \tilde{R})(X, \xi)\xi \\ &+ \eta(X)(\tilde{\nabla}_W \tilde{R})(\xi, Y)Z + \eta(X)\eta(Z)(\tilde{\nabla}_W \tilde{R})(\xi, Y)Z. \end{aligned}$$

Using (2.36) in (3.19) we get

$$\begin{aligned}
(3.20) \quad & (\tilde{\nabla}_W \tilde{R})(\phi^2 X, \phi^2 Y) \phi^2 Z \\
&= (\tilde{\nabla}_W \tilde{R})(X, Y) Z - \eta(Z) R(X, Y) \phi W - \eta(Y) R(X, \phi W) Z + \eta(X) R(Y, \phi W) Z \\
&+ \frac{1}{2} [\eta(Z) \{ \Omega(Y, W) X - \Omega(X, W) Y \} + \eta(Y) \Omega(W, Z) X - \eta(X) \Omega(W, Z) Y] \\
&- \eta(Z) \{ \alpha(Y, \phi W) X - \alpha(X, \phi W) Y \} + \eta(Y) \alpha(Z, \phi W) X - \eta(X) \alpha(Z, \phi W) Y \\
&- \eta(X) \{ \alpha(Y, Z) W - \eta(W) \alpha(Y, Z) \xi \} + \eta(Y) \eta(Z) \{ \alpha(X, W) \xi - \alpha(X, \phi W) \xi \} \\
&- \eta(X) \eta(Z) \{ \alpha(Y, W) \xi - \alpha(Y, \phi W) \xi \} - \frac{1}{2} \eta(X) \{ g(Y, Z) W - \eta(W) g(Y, Z) \xi \} \\
&+ \frac{1}{2} \eta(Y) \{ g(X, Z) W - \eta(W) g(X, Z) \xi \}.
\end{aligned}$$

From (3.18) and (3.20), we get

$$\begin{aligned}
(3.21) \quad & (\tilde{\nabla}_{\phi^2 W} \tilde{R})(\phi^2 X, \phi^2 Y) \phi^2 Z \\
&= (\tilde{\nabla}_W \tilde{R})(X, Y) Z - \eta(Z) R(X, Y) \phi W - \eta(Y) R(X, \phi W) Z + \eta(X) R(Y, \phi W) Z \\
&+ \frac{1}{2} [\eta(Z) \{ \Omega(Y, W) X - \Omega(X, W) Y \} + \eta(Y) \Omega(W, Z) X - \eta(X) \Omega(W, Z) Y] \\
&- \eta(Z) \{ \alpha(Y, \phi W) X - \alpha(X, \phi W) Y \} + \eta(Y) \alpha(Z, \phi W) X - \eta(X) \alpha(Z, \phi W) Y \\
&- \eta(X) \{ \alpha(Y, Z) W - \eta(W) \alpha(Y, Z) \xi \} + \eta(Y) \eta(Z) \{ \alpha(X, W) \xi - \alpha(X, \phi W) \xi \} \\
&- \eta(X) \eta(Z) \{ \alpha(Y, W) \xi - \alpha(Y, \phi W) \xi \} - \frac{1}{2} \eta(X) \{ g(Y, Z) W - \eta(W) g(Y, Z) \xi \} \\
&+ \frac{1}{2} \eta(Y) \{ g(X, Z) W - \eta(W) g(X, Z) \xi \}.
\end{aligned}$$

From (3.14) and (3.21) we obtain

$$\begin{aligned}
(3.22) \quad & (\tilde{\nabla}_W \tilde{R})(X, Y) Z \\
&= \left[g(R(X, Y) W, Z) - g(R(X, Y) \phi W, Z) + \alpha(X, W) g(Y, Z) - \alpha(Y, W) g(X, Z) \right. \\
&+ \alpha(Y, Z) g(X, W) - \alpha(X, Z) g(Y, W) + \frac{1}{2} \{ \eta(Y) g(X, W) - \eta(X) g(Y, W) \} \eta(Z) \\
&- \eta(Y) \eta(W) \alpha(X, Z) + \alpha(Y, \phi W) g(X, Z) - \alpha(X, \phi W) g(Y, Z) \\
&+ \Omega(Y, W) \alpha(X, Z) - \Omega(X, W) \alpha(Y, Z) + \frac{1}{2} \{ \eta(X) \Omega(Y, W) - \eta(Y) \Omega(X, W) \} \eta(Z) \\
&+ g(X, W) g(Y, Z) - g(Y, W) g(X, Z) + g(Y, W) \Omega(X, Z) - g(X, W) \Omega(Y, Z) \\
&+ \left. \{ \eta(Y) \Omega(X, Z) - \eta(X) \Omega(Y, Z) \} \eta(W) + \Omega(Y, W) g(X, Z) - \Omega(X, W) g(Y, Z) \right]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ \eta(X)\Omega(Y, W) - \eta(Y)\Omega(X, W) \right\} \eta(Z) + \Omega(X, W)\Omega(Y, Z) - \Omega(Y, W)\Omega(X, Z) \Big] \xi \\
& + \eta(Z)R(X, Y)\phi W + \eta(Y)R(X, \phi W)Z - \eta(X)R(Y, \phi W)Z \\
& - \frac{1}{2} \left[\eta(Z) \{ \Omega(Y, W)X - \Omega(X, W)Y \} + \eta(Y)\Omega(W, Z)X - \eta(X)\Omega(W, Z)Y \right] \\
& + \eta(Z) \{ \alpha(Y, \phi W)X - \alpha(X, \phi W)Y \} - \eta(Y)\alpha(Z, \phi W)X + \eta(X)\alpha(Z, \phi W)Y \\
& + \eta(X)\alpha(Y, Z)W + \frac{1}{2} \{ \eta(X)g(Y, Z)W - \eta(Y)g(X, Z)W \}.
\end{aligned}$$

Thus in a locally ϕ -symmetric LP-Sasakian manifold with respect to semi-symmetric metric connection, the relation (3.22) holds for any $X, Y, Z, W \in \chi(M)$.

Next, if the relation (3.22) holds in an LP-Sasakian manifold with respect to semi-symmetric metric connection then for any horizontal vector fields X, Y, Z, W , we obtain the relation (3.4) and hence the manifold is locally ϕ -symmetric with respect to semi-symmetric metric connection. Thus we can state the following:

Theorem 3.4. *An LP-Sasakian manifold (M^n, g) is locally ϕ -symmetric with respect to semi-symmetric metric connection if and only if the relation (3.22) holds for any vector fields $X, Y, Z, W \in \chi(M)$.*

In view of (2.36), (3.22) yields

$$\begin{aligned}
(3.23) \quad & (\nabla_W R)(X, Y)Z \\
& = [\Omega(W, Y) - g(W, Y)]\eta(Z)X + [\Omega(W, Z) - g(W, Z)]\eta(Y)X \\
& + 2\eta(Z)\eta(W)[\eta(X)Y - \eta(Y)X] + [\alpha(Y, Z)\eta(X) - \alpha(X, Z)\eta(Y)]W \\
& + [g(W, X) - \Omega(W, X)]\eta(Z)Y + [g(W, Z) - \Omega(W, Z)]\eta(X)Y \\
& + g(Y, Z)\eta(X)\left[\frac{1}{2}W - \phi W\right] - g(X, Z)\eta(Y)\left[\frac{1}{2}W - \phi W\right] \\
& + \eta(Z)R(X, Y)\phi W + \eta(Y)R(X, \phi W)Z - \eta(X)R(Y, \phi W)Z \\
& + \frac{1}{2} \left[\eta(X)\Omega(W, Z)Y - \eta(Y)\Omega(W, Z)X - \eta(Z) \{ \Omega(Y, W)X - \Omega(X, W)Y \} \right] \\
& + \eta(Z) \{ \alpha(Y, \phi W)X - \alpha(X, \phi W)Y \} - \eta(Y)\alpha(Z, \phi W)X + \eta(X)\alpha(Z, \phi W)Y \\
& + \eta(X)\alpha(Y, Z)W + \frac{1}{2} \{ \eta(X)g(Y, Z)W - \eta(Y)g(X, Z)W \} \\
& + \left[2 \{ \eta(X)g(Y, Z) - \eta(Y)g(X, Z) \} \eta(Y) - g(R(X, Y)\phi W, Z) \right] \\
& + \frac{1}{2} \{ \eta(Y)g(X, W) - \eta(X)g(Y, W) \} \eta(Z) \\
& - \eta(Y)\eta(W)\alpha(X, Z) + \alpha(Y, \phi W)g(X, Z) - \alpha(X, \phi W)g(Y, Z) + \Omega(Y, W)\alpha(X, Z)
\end{aligned}$$

$$\begin{aligned}
& - \Omega(X, W)\alpha(Y, Z) + \frac{3}{2}\{\eta(X)\Omega(Y, W) - \eta(Y)\Omega(X, W)\}\eta(Z) + g(Y, W)\Omega(X, Z) \\
& - g(X, W)\Omega(Y, Z) + \{\eta(Y)\Omega(X, Z) - \eta(X)\Omega(Y, Z)\}\eta(W) + 2\{\Omega(Y, W)g(X, Z) \\
& - \Omega(X, W)g(Y, Z)\} + \Omega(X, W)\Omega(Y, Z) - \Omega(Y, W)\Omega(X, Z)\Big]\xi.
\end{aligned}$$

This leads to the following:

Theorem 3.5. *In a locally ϕ -symmetric LP-Sasakian manifold with respect to semi-symmetric metric connection the relation (3.23) holds for any vector fields $X, Y, Z, W \in \chi(M)$.*

From (3.8) and (3.23), we can state the following:

Theorem 3.6. *A locally ϕ -symmetric LP-Sasakian manifold is invariant under a semi-symmetric metric connection if and only if the relation*

$$\begin{aligned}
(3.24) \quad & [\Omega(W, Y) - g(W, Y)]\eta(Z)X + [\Omega(W, Z) - g(W, Z)]\eta(Y)X \\
& + 2\eta(Z)\eta(W)[\eta(X)Y - \eta(Y)X] + [\alpha(Y, Z)\eta(X) - \alpha(X, Z)\eta(Y)]W \\
& + [g(W, X) - \Omega(W, X)]\eta(Z)Y + [g(W, Z) - \Omega(W, Z)]\eta(X)Y \\
& + \frac{1}{2}[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)]W + \eta(Z)[g(Y, W)\phi X - g(X, W)\phi Y] \\
& - \frac{1}{2}[\eta(X)\Omega(W, Z)Y - \eta(Y)\Omega(W, Z)X - \eta(Z)\{\Omega(Y, W)X - \Omega(X, W)Y\}] \\
& + \eta(Z)\{\alpha(Y, \phi W)X - \alpha(X, \phi W)Y\} - \eta(Y)\alpha(Z, \phi W)X + \eta(X)\alpha(Z, \phi W)Y \\
& + \eta(X)\alpha(Y, Z)W + \frac{1}{2}\{\eta(X)g(Y, Z)W - \eta(Y)g(X, Z)W\} \\
& + \left[2\{\eta(X)g(Y, Z) - \eta(Y)g(X, Z)\}\eta(Y) + \frac{1}{2}\{\eta(Y)g(X, W) - \eta(X)g(Y, W)\}\eta(Z) \right. \\
& - \eta(Y)\eta(W)\alpha(X, Z) + \alpha(Y, \phi W)g(X, Z) - \alpha(X, \phi W)g(Y, Z) + \Omega(Y, W)\alpha(X, Z) \\
& - \Omega(X, W)\alpha(Y, Z) + \frac{3}{2}\{\eta(X)\Omega(Y, W) - \eta(Y)\Omega(X, W)\}\eta(Z) + g(Y, W)\Omega(X, Z) \\
& - g(X, W)\Omega(Y, Z) + \{\eta(Y)\Omega(X, Z) - \eta(X)\Omega(Y, Z)\}\eta(W) + \Omega(Y, W)g(X, Z) \\
& \left. - \Omega(X, W)g(Y, Z) + \Omega(X, W)\Omega(Y, Z) - \Omega(Y, W)\Omega(X, Z)\right]\xi = 0
\end{aligned}$$

holds for arbitrary vector fields $X, Y, Z, W \in \chi(M)$.

4. LOCALLY ϕ -RICCI SYMMETRIC LP-SASAKIAN MANIFOLDS ADMITTING SEMI-SYMMETRIC CONNECTION

Definition 4.1. An LP-Sasakian manifold M is said to be locally ϕ -Ricci symmetric with respect to semi-symmetric metric connection if it satisfies the condition

$$(4.1) \quad \phi^2((\tilde{\nabla}_X \tilde{Q})(Y)) = 0$$

for horizontal vector fields X and Y , where \tilde{Q} is the Ricci-operator with respect to semi-symmetric metric connection, i.e. $g(\tilde{Q}X, Y) = \tilde{S}(X, Y)$ for all vector fields X, Y .

Let us take an LP-Sasakian manifold, which is ϕ -Ricci symmetric with respect to semi-symmetric metric connection. Then by virtue of (2.1) it follows from (4.1) that

$$(\tilde{\nabla}_X \tilde{Q})(Y) + \eta((\tilde{\nabla}_X \tilde{Q})(Y))\xi = 0$$

from which it follows that

$$(4.2) \quad (\tilde{\nabla}_X \tilde{S})(Y, Z) = 0$$

for all horizontal vector fields X and Y and Z .

Let X, Y, Z be arbitrary vector fields of $\chi(M)$. We now compute $(\tilde{\nabla}_{\phi^2 X} \tilde{S})(\phi^2 Y, \phi^2 Z)$ in two different ways. Since $\phi^2 X, \phi^2 Y, \phi^2 Z$ are horizontal vector fields for all $X, Y, Z \in \chi(M)$, from (4.2) we have

$$(4.3) \quad (\tilde{\nabla}_{\phi^2 X} \tilde{S})(\phi^2 Y, \phi^2 Z) = 0$$

for all $X, Y, Z \in \chi(M)$. By virtue of (2.1) we get

$$(4.4) \quad (\tilde{\nabla}_{\phi^2 X} \tilde{S})(\phi^2 Y, \phi^2 Z) = (\tilde{\nabla}_X \tilde{S})(\phi^2 Y, \phi^2 Z) + \eta(X)(\tilde{\nabla}_\xi \tilde{S})(\phi^2 Y, \phi^2 Z).$$

Now for any horizontal vector fields Y and Z we have from (4.2) that

$$(\tilde{\nabla}_\xi \tilde{S})(Y, Z) = 0,$$

which implies that

$$(4.5) \quad (\tilde{\nabla}_\xi \tilde{S})(\phi^2 Y, \phi^2 Z) = 0$$

for arbitrary vector fields $Y, Z \in \chi(M)$.

Using (4.5) in (4.4) we get

$$(4.6) \quad (\tilde{\nabla}_{\phi^2 X} \tilde{S})(\phi^2 Y, \phi^2 Z) = (\tilde{\nabla}_X \tilde{S})(\phi^2 Y, \phi^2 Z).$$

In view of (2.1), we get

$$(4.7) \quad \begin{aligned} (\tilde{\nabla}_X \tilde{S})(\phi^2 Y, \phi^2 Z) &= (\tilde{\nabla}_X \tilde{S})(Y, Z) + \eta(Y)(\tilde{\nabla}_X \tilde{S})(Z, \xi) \\ &+ \eta(Z)(\tilde{\nabla}_X \tilde{S})(Z, \xi) + \eta(Y)\eta(Z)(\tilde{\nabla}_X \tilde{S})(\xi, \xi). \end{aligned}$$

Using (2.37) in (4.7) we get

$$(4.8) \quad \begin{aligned} (\tilde{\nabla}_X \tilde{S})(\phi^2 Y, \phi^2 Z) &= (\nabla_X S)(Y, Z) - \eta(Z)S(Y, \phi X) \\ &+ \eta(Y)[S(X, Z) - S(Z, \phi X)] + \eta(Y)\alpha(X, Z) \\ &+ [(2n-1)\eta(X) - da(X)]\eta(Y)\eta(Z) \\ &+ (n-1)\eta(Z)\Omega(X, Y) - (n-3)\eta(Y)\Omega(X, Z) \\ &+ (n - \frac{1}{2})\eta(Y)g(X, Z) - da(X)g(Y, Z). \end{aligned}$$

By virtue of (4.3) and (4.8) we obtain from (4.7) that

$$\begin{aligned}
 (4.9) \quad (\nabla_X S)(Y, Z) &= \eta(Z)S(Y, \phi X) - \eta(Y)[S(X, Z) - S(Z, \phi X)] \\
 &- \eta(Y)\alpha(X, Z) - [(2n-1)\eta(X) - da(X)]\eta(Y)\eta(Z) \\
 &- (n-1)\eta(Z)\Omega(X, Y) + (n-3)\eta(Y)\Omega(X, Z) \\
 &- (n - \frac{1}{2})\eta(Y)g(X, Z) + da(X)g(Y, Z).
 \end{aligned}$$

Thus in a locally ϕ -Ricci symmetric LP-Sasakian manifold with respect to semi-symmetric metric connection, the relation (4.9) holds for any $X, Y, Z \in \chi(M)$.

Next if the relation (4.9) holds in an LP-Sasakian manifold with respect to semi-symmetric metric connection then for any horizontal vector fields X, Y, Z with $\text{tr.}\alpha = \text{constant}$, we obtain $(\nabla_X S)(Y, Z) = 0$ and hence the manifold is locally ϕ -Ricci symmetric with respect to semi-symmetric metric connection. Thus we can state the following:

Theorem 4.1. *An LP-Sasakian manifold (M^n, g) is locally ϕ -Ricci symmetric with respect to semi-symmetric metric connection with $\text{tr.}\alpha = \text{constant}$ if and only if the relation (4.9) holds for any vector fields $X, Y, Z \in \chi(M)$.*

Putting $Y = \xi$ in (4.9) and using (2.38), we get

$$\begin{aligned}
 (4.10) \quad S(X, Z) &= 2(n-2)\Omega(X, Z) - \alpha(X, Z) \\
 &- (n - \frac{1}{2})g(X, Z) + (2n-1)\eta(X)\eta(Z)
 \end{aligned}$$

for any vector fields $X, Z \in \chi(M)$.

This leads to the following:

Theorem 4.2. *In a locally ϕ -Ricci symmetric LP-Sasakian manifold with respect to semi-symmetric metric connection, the Ricci tensor is of the form (4.10).*

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